(D) Prove that the intersection of two normal subgroups of a group is a normal subgroup. 6

Question—V

5. (A) Show that $J_1(x) = \frac{x}{2} - \frac{x^3}{2^2.4} + \frac{x^5}{2^2.4^2.6} - \ldots$ 1½

(B) Prove that $\int_{-1}^{1} P_0(x)\,dx = 2$. 1½

(C) Show that $L(1) = 1/s,\ s > 0$. 1½

(D) Find the Laplace transform of $f(t) = (3e^{2t} - 4)^2$. 1½

(E) Solve $y'' + a^2 y = 0$ with $y(0) = 1$ and $y'(0) = 0$, where $y = y(t)$. 1½

(F) Find the Fourier transform of e^{-x}. 1½

(G) Prove that every subgroup of an abelian group is normal. 1½

(H) Let G be the multiplicative group of all positive reals and G′, the additive group of reals. Show that the mapping $f : G \to G'$ defined by $f(x) = \log x,\ \forall\ x \in G$, is homomorphism. 1½

NTK/KW/15/5824

Bachelor of Science (B.Sc.) Semester—III Examination
MATHEMATICS
(M_6–Differential Equations & Group Homomorphism)
Paper—VI

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$. Hence show that $J_n'(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$. 6

(B) Prove that $J_{-1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \cos x$ and $J_{1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$. 6

OR

(C) Prove that $P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. 6

(D) Prove that $\int_{-1}^{1} P_m(x) \cdot P_n(x)\, dx = 0$ if $m \neq n$. 6

UNIT—II

2. (A) If a and b are any constants, f and g are functions of t, t > 0, then prove that :

$$L\,[a\, f(t) + b\, g(t)] = a\, L(f(t)) + b\, L\,(g(t)).$$

Hence find L [sin 3t cos t]. 6

(B) If L (f(t)) = F(s), then prove that $L\,[e^{at} f(t)] = F(s - a)$. Hence find $L\,[e^{-4t}\,(\sin 5t + 3\cos 2t)]$. 6

OR

(C) If $L^{-1}(F(s)) = f(t)$ and $L^{-1}(G(s)) = g(t)$, then prove that :

$$L^{-1}[F(s)G(s)] = \int_0^t f(u)\, g(t-u)\, du.$$ 6

(D) Evaluate $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$ by using convolution property. 6

UNIT—III

3. (A) Solve $x' + 5x + 2y = t$, $y' + 2x + y = 0$, $x(0) = 0$, $y(0) = 0$, where $x = x(t)$, $y = y(t)$. 6

(B) Solve $\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}$, given that u(0, t) = 0, u(5, t) = 0, u(x, 0) = sin πx. 6

OR

(C) Find the Fourier sine transform of $\frac{e^{-\lambda x}}{x}$, $\lambda > 0$. 6

(D) Let u(x, t) be a function defined for t > 0 and $x \in [a, b]$. Show that :

(i) $L\left(\frac{\partial u}{\partial x}\right) = \frac{dU}{dx}$

(ii) $L\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{d^2 U}{dx^2}$

where U = U(x, s) = L [u(x, t)]. 6

UNIT—IV

4. (A) If $f : G \to G'$ be a homomorphism of a group G into a group G′. Then prove that the kernel K of f is a normal subgroup of G. 6

(B) Show that the mapping $f : C \to R$ such that f(x + iy) = x is a homomorphism of the additive group C of complex numbers onto the additive group R of real numbers. Find the kernel of f. 6

OR

(C) Prove that every homomorphic image of a group G is isomorphic to some quotient group of G. 6